NACA TN 3841

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3841

DISTRIBUTION OF NORMAL COMPONENT OF INDUCED VELOCITY IN LATERAL PLANE OF A LIFTING ROTOR

By Walter Castles, Jr., and Howard L. Durham, Jr.

Georgia Institute of Technology

LIBRARY COPY

DEC 26 1956

LANGLEY AERONAUTICAL LABORATOR
LIBRARY NACA
LANGLEY FIELD, VIRGINIA

...REFERENCE
...TO BE TAKEN FROM THIS ROOM

Washington

December 1956

NASA Technical Library
3 1176 01434 0724

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3841

DISTRIBUTION OF NORMAL COMPONENT OF INDUCED VELOCITY IN LATERAL PLANE OF A LIFTING ROTOR

By Walter Castles, Jr., and Howard L. Durham, Jr.

SUMMARY

Values of the nondimensional normal component of induced velocity in the lateral plane of a uniformly loaded lifting rotor are given in the form of graphs and tables. The values were computed by use of the Biot-Savart relation using the assumption that the wake vortex distribution consists of a uniform, semi-infinite elliptic cylinder.

The necessary auxiliary equations are given so that the graphs or tables of the induced velocity ratios may be used to estimate numerical values of the normal component of induced velocity and the associated flow angle in the lateral plane of any given rotor or set of laterally disposed rotors.

INTRODUCTION

The values of the normal component of induced velocity in the longitudinal plane of a lifting rotor were presented in reference 1. It was subsequently found that the results of reference 1, which were computed for a wake vortex system consisting of a uniform, semi-infinite, elliptic cylinder, were in reasonable agreement, for the higher speed flight conditions, with experimentally determined values of the induced velocities and induced angles. (See ref. 2 for example.) [modified by approximation to account for non-uniformity of disk loading]

The present report extends the work of reference 1 to cover the distribution of the values of the normal component of induced velocity in the lateral plane of a lifting rotor. In the analysis of reference 1 and that presented herein it is assumed that the wake vortex distribution corresponds to the limiting case of the wake behind a uniformly loaded actuator disk as the ratio of the normal component of the induced velocity to the free-stream velocity becomes vanishingly small. Since it was found that the first of the double integrations involved in the application of the Biot-Savart relation to the wake vortex system could be performed explicitly and since a digital computer was available for the required second numerical integration, the method used to compute the tables of the present report differs from that of reference 1.

SYMBOLS

a_1	coefficient of cosine term in Fourier series for blade flapping angle measured with respect to plane of zero feathering, or coefficient of $-\sin \psi$ in equation for blade angle measured with respect to plane of rotation where blade angle is $A_o - a_1 \sin \psi + b_1 \cos \psi$
b_1	
C_T	rotor thrust coefficient, $T/\rho\pi\Omega^2R^4$
l	distance along wake measured from rotor disk
m	tangent of wake angle χ
R	rotor radius
R_o	radius of point P from Z- or rotor axis
R'	length of radius vector from vortex element to point P
r_o	nondimensional radius of point $P(X_o,Y_o,Z_o)$ from rotor axis, R_o/R
S	length of vortex filament
T	rotor thrust
v	normal component of induced velocity at center of rotor
V	velocity of helicopter along flight path
V_i	normal component of induced velocity at point P
X,Y,Z	coordinates of a wake vortex element
X_o,Y_o,Z_o	coordinates of point $P(X_o,Y_o,Z_o)$

y slope of linear approximation to curve for nondimensional normal component of induced velocity on lateral diameter of rotor as used to obtain value of $V_i/\Omega R$ where $\left(V_i/\Omega R\right) = (-v/\Omega R) + yx \sin \psi + wx \cos \psi$

$y_o = Y_o/R$

$z_o = Z_o/R$

α angle of attack of rotor plane of zero feathering

α_v angle of attack of rotor plane of rotation

$\alpha_{y_o z_o}$ flow angle at a point in YZ- or lateral plane measured between resultant of longitudinal and vertical component of local velocity and plane parallel to rotor plane of rotation

$(\alpha_i)_{y_o z_o}$ induced flow angle at point in lateral plane, $\tan^{-1}\left(V_i/V \cos \alpha_v\right)$

γ strength of vortex sheet per unit length along wake

θ azimuth angle of wake vortex element measured from upwind or negative X-direction

$\lambda = (V \sin \alpha - v)/\Omega R$

$\lambda_v = (V \sin \alpha_v - v)/\Omega R$

$\mu = V \cos \alpha/\Omega R$

$\mu_v = V \cos \alpha_v/\Omega R$

ρ air density

χ wake angle measured between positive Z- or rotor axis and axis of wake

ψ azimuth angle of point $P\left(X_o, Y_o, Z_o\right)$ measured from downwind or positive X-direction

Ω angular velocity of rotor blades

ANALYSIS

Assuming that the wake vortex distribution consists of a uniform semi-infinite elliptic cylinder composed of ring vortex elements lying in planes parallel to the rotor disk, the ratio V_i/v of the normal component of induced velocity at a point $P(X_o,Y_o,Z_o)$ to that at the center of the rotor is (the derivation is given in the appendix)

$$\frac{V_i}{v} = \frac{1}{2\pi}\int_0^{2\pi} \frac{A - B\sqrt{C}}{\sqrt{C}\left(\sqrt{C} - D\right)}\,d\theta \tag{1}$$

where the coordinate system is as shown in figure 1 and

$$A = 1 + r_o \cos(\psi - \theta)$$

$$B = m \cos\theta \Big/ \sqrt{1 + m^2}$$

$$C = 1 + r_o^2 + z_o^2 + 2r_o \cos(\psi - \theta)$$

$$D = \left(z_o + mr_o \cos\psi + m \cos\theta\right) \Big/ \sqrt{1 + m^2}$$

For points in the XY- or lateral plane of the rotor for which $\psi = 90^o$, equation (1) reduces to

$$\frac{V_i}{v} = \frac{1}{2\pi}\int_0^{2\pi} \frac{A' - B\sqrt{C'}}{\sqrt{C'}\left(\sqrt{C'} - D'\right)}\,d\theta \tag{2}$$

where

$$A' = 1 + r_o \sin\theta$$

$$C' = 1 + r_o^2 + z_o^2 + 2r_o \sin\theta$$

$$D' = B + \left(z_o \Big/ \sqrt{1 + m^2}\right)$$

Since the integral of equation (2) cannot be evaluated in terms of elementary functions, the integration for the present report was carried

out with the assistance of a digital computer setup to use the trapezoidal rule with 180 equally spaced increments in θ. This procedure appears to provide an accuracy of better than ± 1 in the third decimal place of the answers except for a few of the points which lie very close to the wake boundary.

RESULTS

The computed values of the induced velocity ratio V_i/v for a rectangular grid of points lying in the XY- or lateral plane of the rotor and extending two rotor radii above and below and three radii laterally are given in tables 1(a) through 1(e) for wake angles of $\chi = 45^\circ$, 63.43°, 75.97°, 84.29°, and 90°, or $\tan \chi = 1$, 2, 4, 10, and ∞, respectively. Values of V_i/v on the lateral axis of the rotor for these same wake angles are given in table 2.

Figures 2(a) through 2(e) show plots of constant values of V_i/v obtained by interpolation from table 1.

The velocity ratios are symmetrical about the XZ- or longitudinal center plane of the rotor and consequently are given only for one side of the lateral or XY-plane. Similarly, the values of V_i/v are given only for wake angles in the first quadrant, or $0 \leqq \chi \leqq 90^\circ$. The values for wake angles in the second quadrant, or $90^\circ \leqq \chi \leqq 180^\circ$, may be obtained by referring to the table or graph for the angle $(180^\circ - \chi)$ and using the value of V_i/v for the point having the negative of the Z-coordinate of the point in question (i.e., on the opposite side of the rotor).

The values of the induced velocity ratio V_i/v on the lateral axis of the rotor are shown in figure 3. It is seen that the assumption of a uniform skewed cylinder for the wake vortex structure gives a constant value for the normal component of the induced velocity on the lateral rotor diameter.

APPLICATION OF RESULTS

Following the procedure given in reference 1 the value of the normal component of selfinduced velocity v at the center of a rotor can be calculated from the equation

$$v \approx \frac{\frac{1}{2}\Omega R C_T}{\left(1 - \frac{3}{2}\mu_v^2\right)\sqrt{\mu_v^2 + \lambda_v^2}} \approx \frac{\frac{1}{2}\Omega R C_T}{\left(1 - \frac{3}{2}\mu^2\right)\sqrt{\lambda^2 + \mu^2}} \tag{3}$$

and the associated wake angle from the equation

$$\chi = \tan^{-1}\left(\frac{-\mu_v}{\lambda_v}\right)$$

$$= \tan^{-1}\left(\frac{-\mu}{\lambda}\right) + a_1 \tag{4}$$

for $\chi < 90^\circ$, or $\lambda_v = \lambda \cos a_1 + \mu \sin a_1 < 0$, or the equation

$$\chi = \cot^{-1}\left(\frac{-\mu_v}{\lambda_v}\right)$$

$$= \cot^{-1}\left(\frac{-\mu}{\lambda}\right) - a_1 \tag{5}$$

for $\chi > 90^\circ$, or $\lambda_v = \lambda \cos a_1 + \mu \sin a_1 > 0$. The value of V_i/v at a given point $P(y_o, z_o)$ in the lateral plane and thus the value of V_i for the computed values of v and χ can then be obtained by interpolation among the graphs of tables.

The normal component of induced velocity V_i at a point $P(y_o, z_o)$ in the common lateral plane of a set of laterally disposed rotors may be approximated by taking the algebraic sum of the components of V_i at P induced by each individual rotor.

The flow angle $\alpha_{y_o z_o}$ at $P(y_o, z_o)$ measured between the resultant of the local longitudinal and vertical components of velocity and a plane parallel to the tip-path plane is

$$\alpha_{y_o z_o} \approx \tan^{-1}\left[\frac{V \sin \alpha_v - (V_i)_1 - (V_i)_2 - (V_i)_n - \cdots}{V \cos \alpha_v}\right] \tag{6}$$

and the local induced angle is

$$(\alpha_i)_{y_o z_o} \approx - \frac{(V_i)_1 + (V_i)_2 + (V_i)_n + \cdots}{V \cos \alpha_v} \tag{7}$$

where $(V_i)_{1,2,n}$ denotes the increments from the individual rotors of the set.

The variations in the normal component of induced velocity along the lateral diameters of a pair of closely spaced side-by-side rotors are large and will have a significant effect on the equilibrium value of a_1 and the tip-stall-limited top speed. The terms for a linear lateral induced velocity variation of slope y were included in the equations of reference 3 for the equilibrium values of the parameters. Consequently, an estimate may be obtained of the effects of the total lateral induced velocity variation by including the effects of the interference components in the y-terms of the appropriate equations of reference 3.

For a pair of equally loaded, closely spaced rotors rotating with their advancing blades adjacent and operating in the higher speed flight conditions, it follows from the equation for y in reference 3 and from the slope of the V_i/y distribution between the points at $R_o/R = 1.2$ and 2.8 in figure 2(d) that

$$y \approx C_T + \frac{v}{2\Omega R} \tag{8}$$

If the retreating blades are on adjacent sides of the rotors, the interference gradient is in the opposite direction and

$$y \approx C_T - \frac{v}{2\Omega R} \tag{9}$$

For many purposes, the increment or decrement in the equilibrium value of a_1 may be obtained with sufficient accuracy from the relevant terms of equation (15) of reference 3 and from equations (3) and (8) or (9) above as

$$\Delta a_1 \approx \frac{y}{\left(1 - \frac{4}{3}\mu_v\right)^2} \approx \frac{C_T}{\left(1 - \frac{4}{3}\mu\right)^2}\left[1 \pm \frac{1}{4\mu\left(1 - \frac{3}{2}\mu^2\right)}\right] \tag{10}$$

CONCLUDING REMARKS

The distributions of the normal component of induced velocity in the lateral plane of a lifting rotor that are given in the present report were computed for a wake consisting of a uniform semi-infinite skewed vortex cylinder. This assumed wake vortex distribution corresponds to the limiting case of the wake behind a uniformly loaded actuator disk as the ratio of the normal component of the induced velocity to the free-stream velocity becomes vanishingly small. Consequently, the results of the present report can be considered to be only a first approximation for the flow about an actual rotor with nonuniform loading operating in the higher speed flight conditions.

For the lower speed flight conditions where the induced velocity is of the same order of magnitude as the free-stream velocity, the discrepancies between the computed values of V_i/v, where V_i is the normal component of the induced velocity at point P and v is the normal component of the induced velocity at the center of the rotor, and values for an actual rotor are probably larger than for the higher speed flight conditions. Some judgment should therefore be exercised in the engineering application of the results of this report.

Georgia Institute of Technology,
Atlanta, Ga., March 9, 1956.

APPENDIX

DETERMINATION OF V_i/v

It is shown in reference 4 that the three components of velocity u, v, and w induced in the X-, Y-, and Z-directions at a point $P(X_o,Y_o,Z_o)$ by a single isolated reentrant vortex filament of strength k and length S are

$$u = \frac{k}{4\pi}\int_0^S \left(\frac{dY}{dS}\frac{Z_o - Z}{R'} - \frac{dZ}{dS}\frac{Y_o - Y}{R'}\right)\frac{dS}{(R')^2} \tag{A1}$$

$$v = \frac{k}{4\pi}\int_0^S \left(\frac{dZ}{dS}\frac{X_o - X}{R'} - \frac{dX}{dS}\frac{Z_o - Z}{R'}\right)\frac{dS}{(R')^2} \tag{A2}$$

$$w = \frac{k}{4\pi}\int_0^S \left(\frac{dX}{dS}\frac{Y_o - Y}{R'} - \frac{dY}{dS}\frac{X_o - X}{R'}\right)\frac{dS}{(R')^2} \tag{A3}$$

where

dS element of vortex filament with coordinates X, Y, and Z

R' length of radius vector from dS to point $P(X_o,Y_o,Z_o)$

Referring to figure 1, which shows the geometry of the assumed wake vortex distribution, it is seen that for an elementary vortex ring of width dl located at a distance Z below the rotor

$$X = mZ - R\cos\theta$$

$$Y = R\sin\theta$$

$$dS = R\,d\theta$$

where

$m = \tan \chi$

χ wake angle

R rotor or wake radius

θ azimuth angle of vortex element dS measured from upwind position

and

$$X_o = R_o \cos \psi$$

$$Y_o = -R_o \sin \psi$$

where

R_o radius of point $P(X_o, Y_o, Z_o)$ from Z- or rotor axis

ψ azimuth angle of point P measured from downwind or positive X-direction

R' length of radius vector from vortex element dS to point P, that is, $(R')^2 = (X - X_o)^2 + (Y - Y_o)^2 + (Z - Z_o)^2$

$l = Z \sec \chi = Z\sqrt{1 + m^2}$

Thus, from the geometry and equation (A3), the Z-component of velocity w induced at $P(X_o, Y_o, Z_o)$ by the wake vortex element of strength $\gamma\, dl$ is

$$w = \frac{\gamma\, dl}{4\pi} \int_0^{2\pi} \left[\frac{(X_o - X)\cos\theta - (Y_o - Y)\sin\theta}{(R')^3} \right] R\, d\theta \qquad \text{(A4)}$$

where γ is the line integral about a unit length of the wake vortex sheet taken in a path parallel to the wake axis; the sign has been reversed so that w is positive for points inside the circular axis of the ring.

Summing up the increments in velocity induced at $P(X_o,Y_o,Z_o)$ by the wake vortex elements, the resultant normal component of induced velocity V_i at P is

$$V_i = \frac{\gamma}{4\pi}\int_0^{\infty}\int_0^{2\pi}\left[\frac{(X_o - X)\cos\theta - (Y_o - Y)\sin\theta}{(R')^3}\right]R\,d\theta\,dl \qquad (A5)$$

Upon making the previously listed substitutions for X_o, X, Y_o, Y, R', and l and dividing all lengths by R, equation (A5) may be reduced to

$$V_i = \frac{\gamma\sqrt{c}}{4\pi}\int_0^{\infty}\int_0^{2\pi}\frac{(A + az)d\theta\,dz}{(C + bz + cz^2)^{3/2}} \qquad (A6)$$

where

$A = 1 + r_o\cos(\psi - \theta)$

$C = 1 + r_o^2 + z_o^2 + 2r_o\cos(\psi - \theta)$

$a = -m\cos\theta$

$b = -2(z_o + r_o m\cos\psi + m\cos\theta)$

$c = 1 + m^2$

$r_o = R_o/R$

$z_o = Z_o/R$

Performing the integration with respect to Z gives

$$V_i = \frac{\gamma}{4\pi}\int_0^{2\pi}\frac{A - B\sqrt{C}}{\sqrt{C}(\sqrt{C} - D)}\,d\theta \qquad (A7)$$

where

$$B = m \cos \theta \Big/ \sqrt{1 + m^2}$$

$$D = \left(z_o + m r_o \cos \psi + m \cos \theta\right) \Big/ \sqrt{1 + m^2}$$

By setting $r_o = z_o = 0$ in equation (A7), it can be shown that the value of the normal component of induced velocity v at the center of the rotor is

$$v = \frac{1}{2} \gamma \tag{A8}$$

Dividing the value of V_i given by equation (A7) by the value of v given by equation (A8) yields equation (1) for the induced velocity ratio V_i/v.

REFERENCES

1. Castles, Walter, Jr., and De Leeuw, Jacob Henri: The Normal Component of the Induced Velocity in the Vicinity of a Lifting Rotor and Some Examples of Its Application. NACA Rep. 1184, 1954. (Supersedes NACA TN 2912.)

2. Heyson, Harry H.: Analysis and Comparison With Theory of Flow-Field Measurements Near a Lifting Rotor in the Langley Full-Scale Tunnel. NACA TN 3691, 1956.

3. Castles, Walter, Jr., and New, Noah C.: A Blade-Element Analysis for Lifting Rotors That Is Applicable for Large Inflow and Blade Angles and Any Reasonable Blade Geometry. NACA TN 2656, 1952.

4. Lamb, Horace: Hydrodynamics. Sixth ed., Dover Pub., 1945, p. 211.

TABLE 1.- NONDIMENSIONAL VALUES OF NORMAL COMPONENT OF INDUCED VELOCITY V_i/v

IN LATERAL PLANE OF LIFTING ROTOR

(a) $\chi = \tan^{-1}1$

Z_o/R	V_i/v for values of R_o/R of -																
	0	0.1	0.3	0.5	0.7	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
-2.0	0.106	0.105	0.103	0.098	0.091	0.083	0.079	0.070	0.061	0.052	0.044	0.037	0.031	0.025	0.021	0.017	0.014
-1.8	.126	.125	.122	.116	.107	.096	.090	.078	.067	.056	.046	.038	.031	.025	.020	.016	.012
-1.6	.152	.151	.147	.138	.126	.111	.104	.088	.073	.059	.048	.038	.030	.023	.018	.014	.010
-1.4	.186	.185	.179	.167	.150	.130	.119	.099	.079	.062	.048	.036	.027	.020	.015	.011	.008
-1.2	.232	.231	.222	.205	.181	.153	.138	.110	.084	.063	.046	.033	.023	.016	.011	.007	.004
-1.0	.293	.291	.279	.255	.221	.181	.161	.121	.087	.060	.041	.027	.017	.010	.006	.002	.000
-.8	.375	.373	.357	.324	.276	.217	.187	.130	.084	.052	.030	.017	.008	.002	-.001	-.003	-.005
-.6	.486	.483	.463	.421	.353	.264	.217	.131	.070	.034	.013	.002	-.005	-.008	-.010	-.010	-.010
-.4	.629	.626	.606	.558	.471	.332	.250	.111	.035	.001	-.013	-.019	-.020	-.020	-.019	-.018	-.016
-.2	.804	.802	.788	.753	.672	.464	.285	.025	-.041	-.051	-.048	-.043	-.038	-.033	-.029	-.026	-.023
0	1.000	1.000	1.000	1.000	1.000	1.000	-----	-.238	-.159	-.115	-.088	-.069	-.056	-.047	-.039	-.034	-.029
.2	1.196	1.198	1.212	1.247	1.328	1.539	-.984	-.461	-.261	-.171	-.123	-.093	-.073	-.059	-.049	-.041	-.035
.4	1.371	1.374	1.395	1.442	1.530	2.184	-.698	-.455	-.295	-.202	-.145	-.110	-.086	-.069	-.056	-.047	-.040
.6	1.515	1.517	1.537	1.580	1.650	-.453	-.453	-.372	-.278	-.205	-.154	-.119	-.094	-.076	-.062	-.052	-.044
.8	1.625	1.627	1.645	1.688	-.138	-.270	-.295	-.283	-.237	-.190	-.150	-.120	-.097	-.079	-.066	-.055	-.047
1.0	-----	-----	.190	.081	-.057	-.162	-.191	-.208	-.193	-.166	-.138	-.115	-.095	-.079	-.067	-.057	-.049
1.2	.232	-----	.174	.085	-.014	-.095	-.122	-.150	-.151	-.139	-.123	-.106	-.090	-.077	-.066	-.057	-.049
1.4	.186	.181	.145	.082	.010	-.053	-.076	-.106	-.117	-.114	-.106	-.095	-.083	-.073	-.064	-.055	-.049
1.6	.152	.148	.123	.077	.024	-.025	-.045	-.074	-.088	-.092	-.089	-.083	-.075	-.067	-.060	-.053	-.047
1.8	.126	.123	.104	.071	.031	-.008	-.024	-.050	-.066	-.073	-.074	-.071	-.067	-.061	-.056	-.050	-.045
2.0	.106	.104	.090	.065	.034	.004	-.010	-.032	-.048	-.057	-.060	-.061	-.058	-.055	-.051	-.047	-.043

TABLE 1.- NONDIMENSIONAL VALUES OF NORMAL COMPONENT OF INDUCED VELOCITY V_i/v

IN LATERAL PLANE OF LIFTING ROTOR - Continued

(b) $X = \tan^{-1}2$

Z_o/R	V_i/v for values of R_o/R of -																
	0	0.1	0.3	0.5	0.7	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
-2.0	0.106	0.105	0.102	0.097	0.089	0.080	0.075	0.064	0.054	0.044	0.036	0.028	0.022	0.016	0.012	0.008	0.005
-1.8	.126	.125	.121	.114	.103	.091	.084	.071	.058	.046	.036	.027	.019	.014	.009	.005	.003
-1.6	.152	.151	.146	.136	.121	.105	.096	.078	.061	.047	.034	.024	.016	.010	.005	.002	-.001
-1.4	.186	.185	.178	.163	.144	.121	.109	.085	.063	.045	.031	.019	.011	.005	.000	-.003	-.005
-1.2	.232	.230	.220	.200	.172	.140	.123	.091	.063	.041	.024	.012	.003	-.002	-.006	-.009	-.010
-1.0	.293	.291	.277	.248	.209	.163	.139	.095	.058	.031	.012	.000	-.007	-.012	-.014	-.015	-.016
-.8	.375	.373	.354	.315	.258	.190	.155	.092	.044	.013	-.006	-.016	-.021	-.023	-.023	-.023	-.022
-.6	.486	.483	.459	.409	.329	.224	.169	.075	.014	-.018	-.032	-.037	-.037	-.036	-.034	-.031	-.029
-.4	.629	.626	.601	.545	.440	.272	.175	.022	-.047	-.069	-.068	-.063	-.057	-.050	-.045	-.040	-.035
-.2	.804	.802	.786	.743	.643	.378	.153	-.129	-.157	-.135	-.111	-.092	-.077	-.065	-.056	-.048	-.042
0	1.000	1.000	1.000	1.000	1.000	1.000	------	-.500	-.300	-.206	-.152	-.118	-.095	-.078	-.065	-.055	-.048
.2	1.196	1.198	1.215	1.257	1.358	1.424	-1.154	-.648	-.376	-.249	-.179	-.136	-.108	-.087	-.072	-.061	-.052
.4	1.371	1.373	1.401	1.460	.045	-.376	-.491	-.463	-.341	-.247	-.184	-.143	-.113	-.092	-.077	-.065	-.055
.6	.486	.480	.415	.276	.064	-.159	-.236	-.289	-.261	-.213	-.171	-.137	-.112	-.093	-.078	-.066	-.057
.8	.375	.369	.318	.218	.079	-.059	-.113	-.174	-.185	-.169	-.146	-.124	-.105	-.089	-.076	-.065	-.056
1.0	.293	.288	.250	.179	.086	-.007	-.046	-.101	-.125	-.127	-.119	-.106	-.094	-.082	-.071	-.062	-.055
1.2	.232	.228	.201	.151	.086	.020	-.009	-.055	-.081	-.092	-.093	-.088	-.081	-.073	-.065	-.058	-.052
1.4	.186	.184	.164	.128	.082	.034	.012	-.025	-.050	-.064	-.070	-.071	-.068	-.063	-.058	-.053	-.048
1.6	.152	.150	.136	.110	.077	.041	.024	-.006	-.028	-.043	-.052	-.055	-.055	-.054	-.051	-.047	-.044
1.8	.126	.125	.114	.095	.071	.044	.031	.007	-.013	-.027	-.037	-.042	-.044	-.045	-.043	-.041	-.039
2.0	.106	.105	.097	.083	.065	.044	.034	.014	-.003	-.016	-.025	-.031	-.035	-.036	-.037	-.036	-.035

TABLE 1.- NONDIMENSIONAL VALUES OF NORMAL COMPONENT OF INDUCED VELOCITY V_i/v

IN LATERAL PLANE OF LIFTING ROTOR - Continued

(c) $\chi = \tan^{-1}\frac{1}{4}$

Z_o/R	V_i/v for values of R_o/R of -																
	0	0.1	0.3	0.5	0.7	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
-2.0	0.106	0.105	0.102	0.095	0.086	0.076	0.070	0.058	0.047	0.036	0.027	0.019	0.013	0.007	0.003	-0.000	-0.003
-1.8	.126	.125	.121	.112	.100	.086	.078	.063	.049	.036	.025	.016	.009	.003	-.001	-.004	-.006
-1.6	.152	.151	.145	.133	.116	.097	.087	.068	.049	.034	.021	.011	.003	-.002	-.006	-.009	-.011
-1.4	.186	.185	.176	.160	.137	.111	.097	.071	.048	.029	.014	.003	-.004	-.010	-.013	-.015	-.016
-1.2	.232	.230	.218	.195	.163	.126	.107	.072	.042	.019	.003	-.008	-.015	-.019	-.021	-.021	-.022
-1.0	.293	.291	.274	.241	.196	.143	.116	.068	.030	.003	-.014	-.023	-.028	-.030	-.030	-.030	-.028
-.8	.375	.373	.350	.305	.239	.161	.121	.054	.005	-.023	-.037	-.043	-.044	-.043	-.040	-.037	-.034
-.6	.486	.482	.454	.396	.302	.181	.119	.018	-.040	-.064	-.070	-.068	-.063	-.057	-.051	-.046	-.041
-.4	.629	.625	.597	.530	.405	.205	.094	-.067	-.122	-.124	-.111	-.096	-.083	-.071	-.062	-.054	-.047
-.2	.804	.802	.782	.732	.609	.275	-.003	-.281	-.252	-.197	-.154	-.123	-.101	-.084	-.071	-.061	-.053
0	1.000	1.000	1.000	1.000	1.000	1.000	-----	-.699	-.387	-.258	-.187	-.144	-.114	-.093	-.078	-.066	-.057
.2	1.196	1.197	1.221	1.283	.394	-.340	-.758	-.640	-.400	-.271	-.198	-.151	-.120	-.098	-.081	-.069	-.059
.4	.629	.623	.576	.459	.232	-.107	-.253	-.354	-.303	-.236	-.183	-.145	-.117	-.097	-.081	-.069	-.060
.6	.486	.480	.434	.336	.180	-.006	-.086	-.182	-.199	-.180	-.153	-.128	-.108	-.091	-.078	-.067	-.058
.8	.375	.371	.334	.260	.155	.040	-.012	-.087	-.121	-.127	-.119	-.107	-.094	-.082	-.071	-.063	-.055
1.0	.293	.289	.262	.209	.137	.060	.025	-.033	-.068	-.084	-.088	-.084	-.078	-.071	-.064	-.057	-.051
1.2	.232	.229	.209	.172	.122	.068	.042	-.002	-.033	-.052	-.061	-.064	-.062	-.059	-.055	-.050	-.046
1.4	.186	.184	.170	.143	.108	.069	.050	.016	-.011	-.029	-.040	-.046	-.048	-.048	-.046	-.043	-.041
1.6	.152	.151	.140	.121	.196	.067	.053	.026	.004	-.013	-.025	-.032	-.036	-.038	-.038	-.037	-.035
1.8	.126	.125	.117	.103	.085	.063	.052	.032	.013	-.002	-.013	-.021	-.026	-.029	-.030	-.030	-.030
2.0	.106	.105	.099	.089	.075	.059	.050	.034	.019	.006	-.004	-.012	-.017	-.021	-.023	-.024	-.025

TABLE 1.- NONDIMENSIONAL VALUES OF NORMAL COMPONENT OF INDUCED VELOCITY V_i/v

IN LATERAL PLANE OF LIFTING ROTOR - Continued

(d) $\chi = \tan^{-1}10$

Z_o/R	V_i/v for values of R_o/R of -																
	0	0.1	0.3	0.5	0.7	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
-2.0	0.106	0.105	0.101	0.094	0.084	0.072	0.066	0.053	0.041	0.029	0.020	0.012	0.005	0.000	-0.004	-0.007	-0.009
-1.8	.126	.125	.120	.110	.097	.081	.073	.056	.041	.027	.016	.007	.000	-.005	-.009	-.011	-.013
-1.6	.152	.151	.144	.130	.112	.091	.080	.058	.039	.023	.010	.000	-.007	-.012	-.015	-.017	-.018
-1.4	.186	.185	.175	.156	.131	.102	.087	.059	.034	.015	.000	-.010	-.016	-.020	-.022	-.023	-.023
-1.2	.232	.230	.216	.190	.154	.114	.093	.055	.024	.015	-.014	-.023	-.028	-.030	-.031	-.031	-.029
-1.0	.293	.291	.271	.235	.183	.125	.096	.045	.006	-.019	-.034	-.041	-.043	-.042	-.041	-.038	-.036
-.8	.375	.372	.347	.296	.222	.135	.093	.021	-.027	-.052	-.061	-.063	-.060	-.056	-.051	-.047	-.042
-.6	.486	.482	.450	.383	.277	.142	.074	-.031	-.083	-.099	-.097	-.089	-.079	-.070	-.062	-.055	-.048
-.4	.629	.625	.593	.516	.372	.142	.019	-.142	-.178	-.163	-.138	-.116	-.098	-.083	-.071	-.062	-.054
-.2	.804	.801	.779	.721	.576	.169	-.156	-.399	-.314	-.233	-.177	-.140	-.113	-.093	-.078	-.067	-.058
0	1.000	1.000	1.000	1.000	1.000	1.000	-----	-.789	-.422	-.277	-.200	-.153	-.121	-.099	-.082	-.070	-.060
.2	.804	.801	.774	.669	.504	-.062	-.461	-.559	-.379	-.265	-.196	-.152	-.121	-.099	-.083	-.070	-.060
.4	.629	.624	.584	.489	.305	.020	-.120	-.262	-.255	-.211	-.169	-.137	-.113	-.094	-.079	-.068	-.059
.6	.486	.481	.442	.360	.229	.068	-.008	-.112	-.150	-.148	-.132	-.114	-.098	-.084	-.073	-.064	-.056
.8	.375	.371	.340	.278	.189	.087	.040	-.036	-.079	-.095	-.095	-.089	-.081	-.072	-.064	-.057	-.051
1.0	.293	.290	.267	.222	.161	.093	.060	.004	-.034	-.055	-.064	-.066	-.064	-.060	-.055	-.050	-.046
1.2	.232	.230	.213	.181	.138	.091	.067	.026	-.006	-.028	-.040	-.046	-.048	-.047	-.045	-.043	-.040
1.4	.186	.185	.173	.150	.120	.085	.068	.037	.011	-.009	-.022	-.030	-.034	-.036	-.036	-.035	-.034
1.6	.152	.151	.142	.126	.104	.079	.066	.042	.021	.004	-.009	-.017	-.023	-.026	-.028	-.028	-.028
1.8	.126	.125	.119	.107	.091	.072	.062	.044	.027	.012	.001	-.008	-.014	-.018	-.021	-.022	-.023
2.0	.106	.105	.100	.092	.079	.065	.058	.043	.029	.017	.007	-.001	-.007	-.011	-.015	-.017	-.018

TABLE 1.- NONDIMENSIONAL VALUES OF NORMAL COMPONENT OF INDUCED VELOCITY V_i/v

IN LATERAL PLANE OF LIFTING ROTOR - Concluded

(e) $X = \tan^{-1}\infty$

Z_o/R	V_i/v (symmetric across $Z_o/R = 0$) for values of R_o/R of -																
	0	0.1	0.3	0.5	0.7	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
-2.0	0.106	0.105	0.101	0.093	0.082	0.069	0.062	0.049	0.036	0.024	0.014	0.006	-0.001	-0.005	-0.009	-0.012	-0.013
-1.8	.126	.125	.119	.109	.094	.077	.068	.050	.034	.020	.009	-.000	-.007	-.011	-.015	-.017	-.018
-1.6	.152	.151	.143	.128	.108	.085	.074	.051	.031	.014	.001	-.008	-.015	-.019	-.021	-.023	-.023
-1.4	.186	.185	.174	.153	.126	.094	.079	.049	.023	.004	-.010	-.020	-.025	-.028	-.029	-.029	-.029
-1.2	.232	.230	.215	.186	.147	.103	.081	.042	.010	-.012	-.026	-.034	-.038	-.039	-.038	-.037	-.035
-1.0	.293	.290	.269	.229	.173	.110	.080	.026	-.013	-.037	-.049	-.053	-.053	-.051	-.048	-.045	-.041
-.8	.375	.372	.344	.288	.207	.113	.068	-.006	-.052	-.073	-.078	-.076	-.071	-.065	-.058	-.052	-.047
-.6	.485	.481	.447	.373	.256	.108	.037	-.069	-.116	-.123	-.115	-.102	-.089	-.078	-.068	-.060	-.053
-.4	.629	.624	.589	.504	.342	.087	-.045	-.200	-.217	-.188	-.155	-.127	-.106	-.089	-.076	-.065	-.057
-.2	.804	.801	.777	.712	.545	.070	-.293	-.482	-.351	-.252	-.189	-.147	-.118	-.097	-.081	-.069	-.060
0	1.000	1.000	1.000	1.000	1.000	1.000	-----	-.809	-.429	-.281	-.203	-.155	-.123	-.100	-.083	-.071	-.061

TABLE 2.- VALUES OF V_i/v ON LATERAL AXIS OF ROTOR

$[Z_o/R = 0]$

R_o/R	V_i/v for values of m. of -				
	1	2	4	10	∞
0→1	1.000	1.000	1.000	1.000	1.000
1.05	-.357	-.911	-1.616	-2.136	-2.284
1.10	-.306	-.718	-1.122	-1.347	-1.401
1.2	-.238	-.500	-.699	-.789	-.809
1.4	-.159	-.300	-.387	-.422	-.429
1.6	-.115	-.206	-.258	-.277	-.281
1.8	-.088	-.152	-.187	-.200	-.203
2.0	-.069	-.118	-.144	-.153	-.155
2.2	-.056	-.095	-.114	-.121	-.123
2.4	-.047	-.078	-.093	-.099	-.100
2.6	-.039	-.065	-.078	-.082	-.083
2.8	-.034	-.055	-.066	-.070	-.071
3.0	-.029	-.048	-.057	-.060	-.061

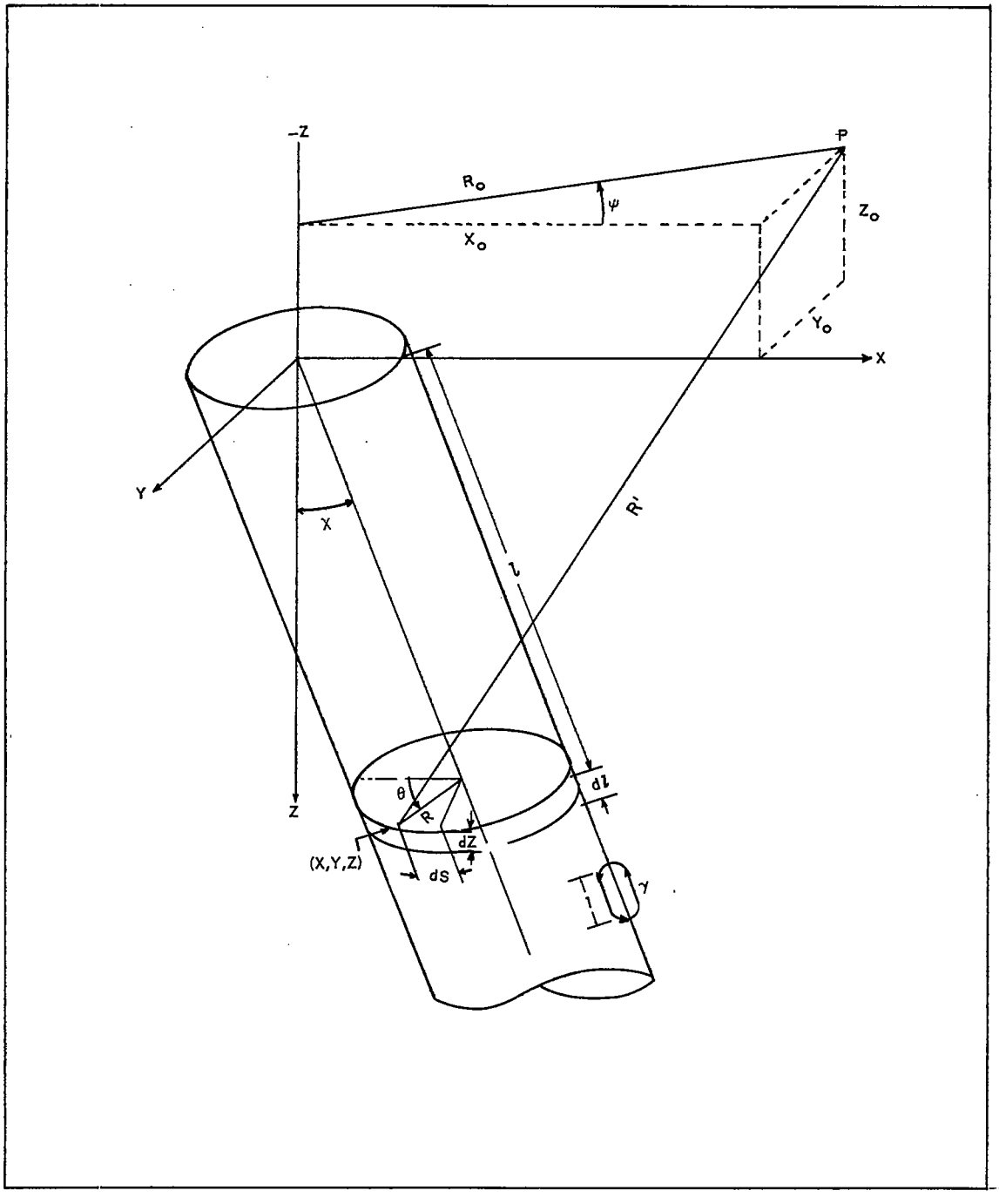
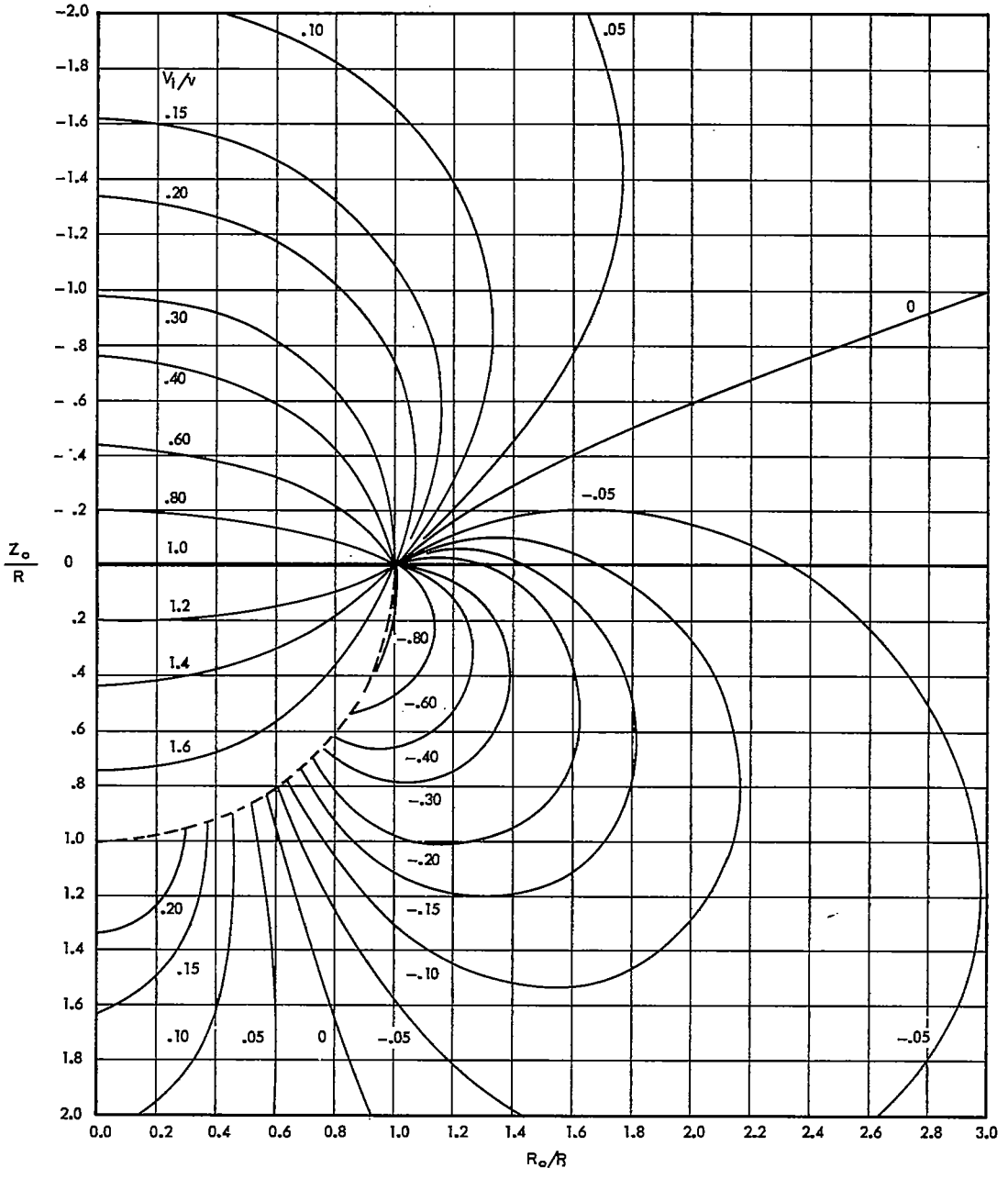
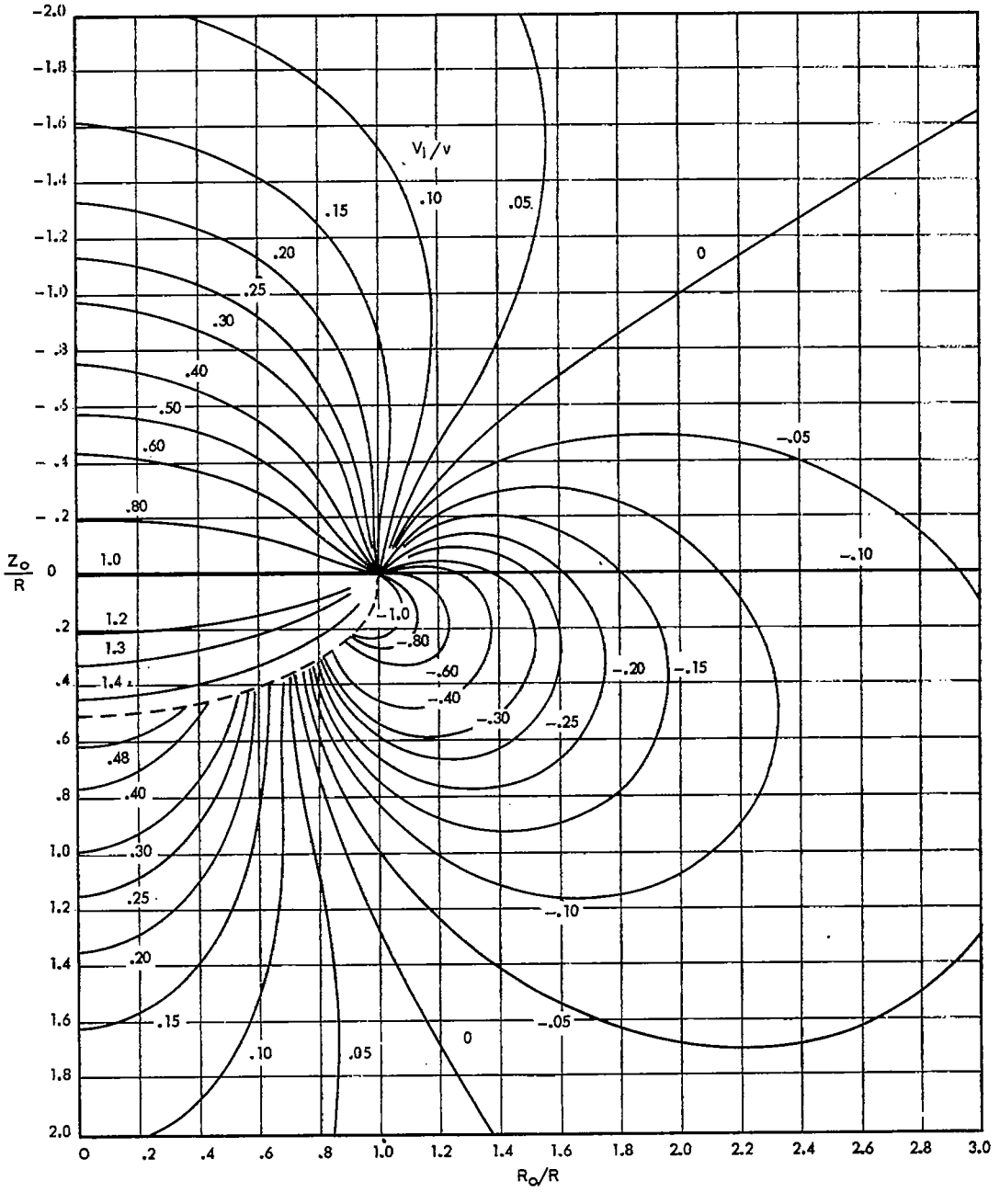


Figure 1.- Geometry of wake vortex system.



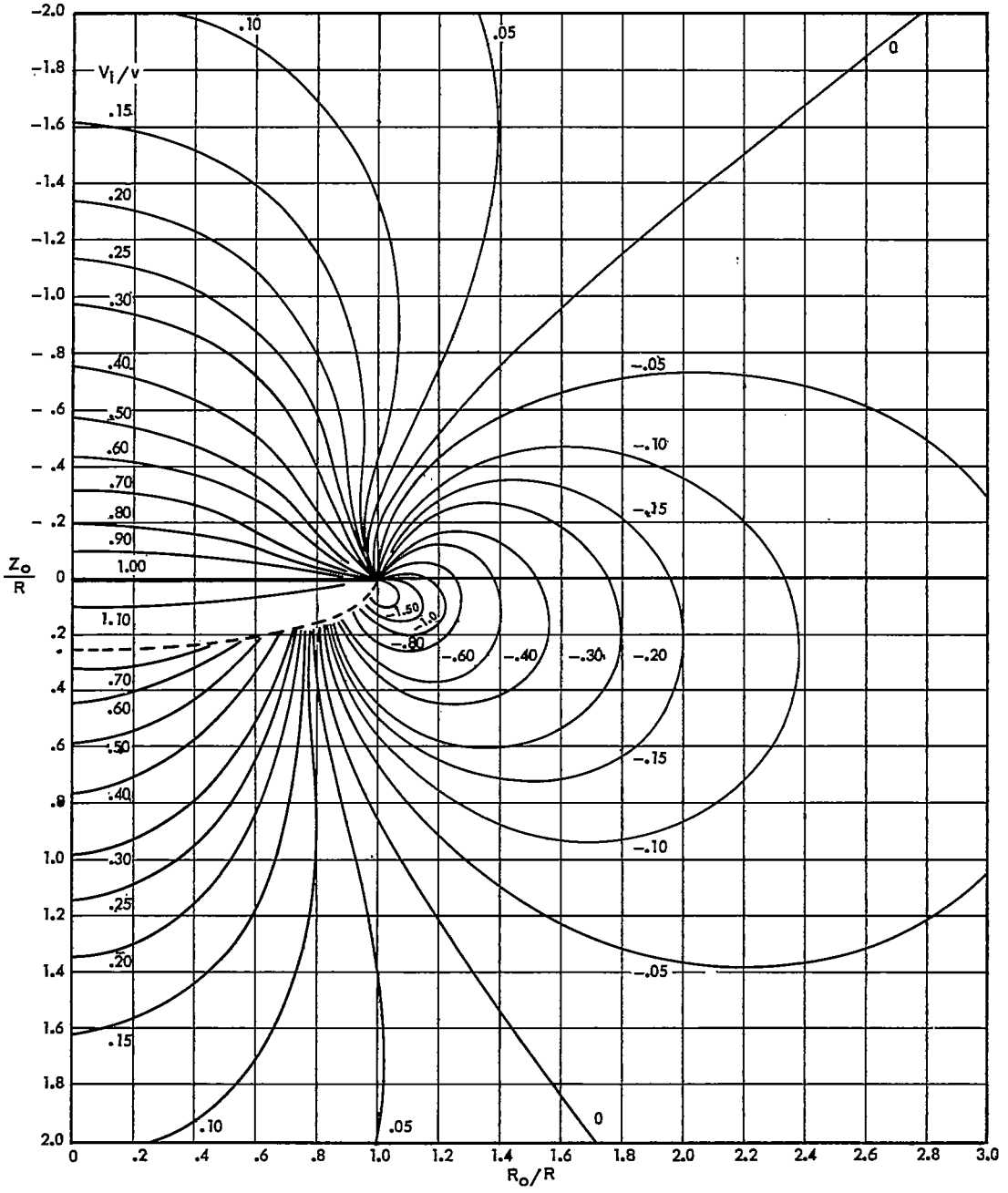
(a) Wake angle = 45° or $\chi = \tan^{-1}1$.

Figure 2.- Lines of constant values of nondimensional normal component of induced velocity V_i/v in lateral plane of lifting rotor.



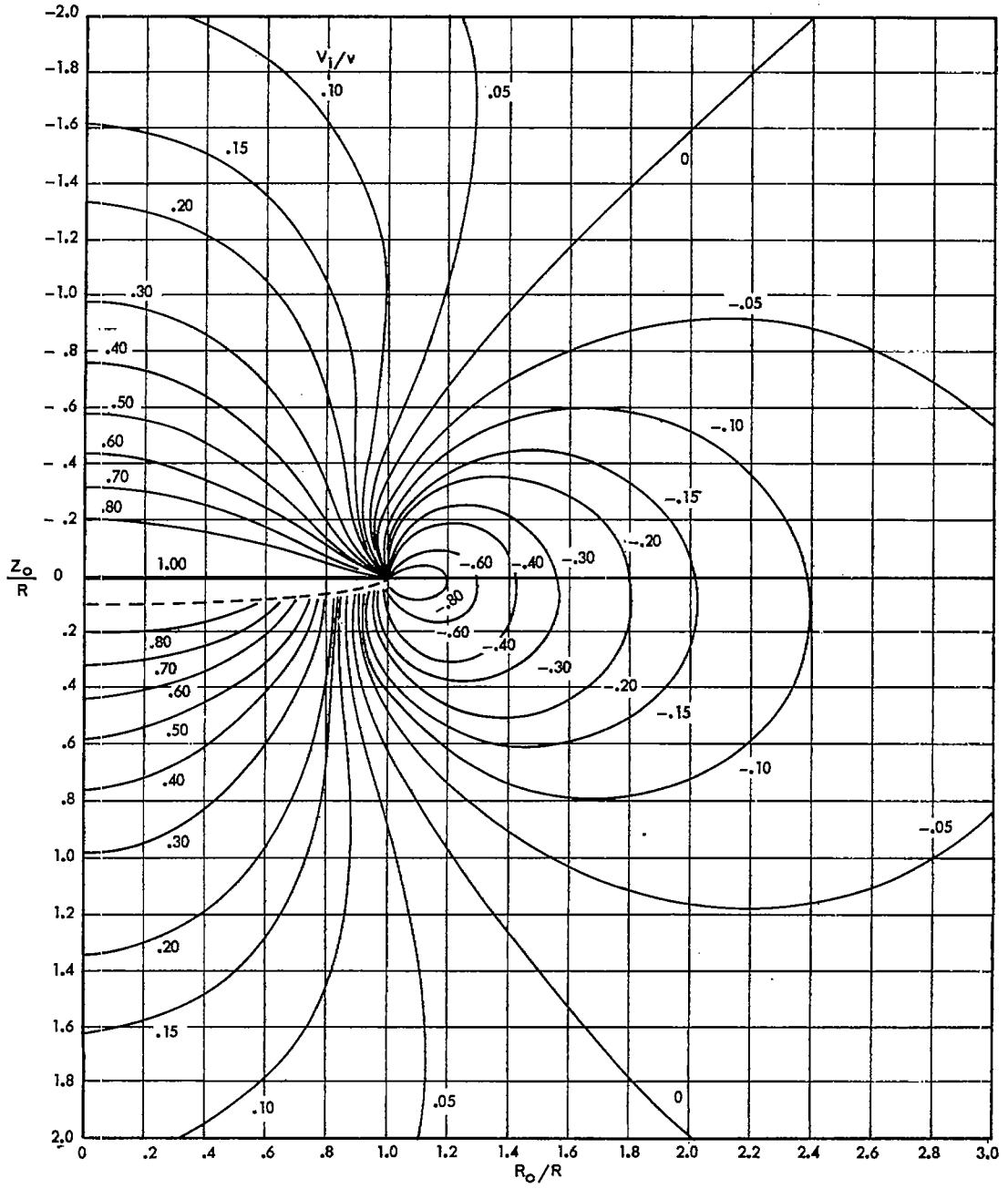
(b) Wake angle = 63.43° or $\chi = \tan^{-1}2$.

Figure 2.- Continued.



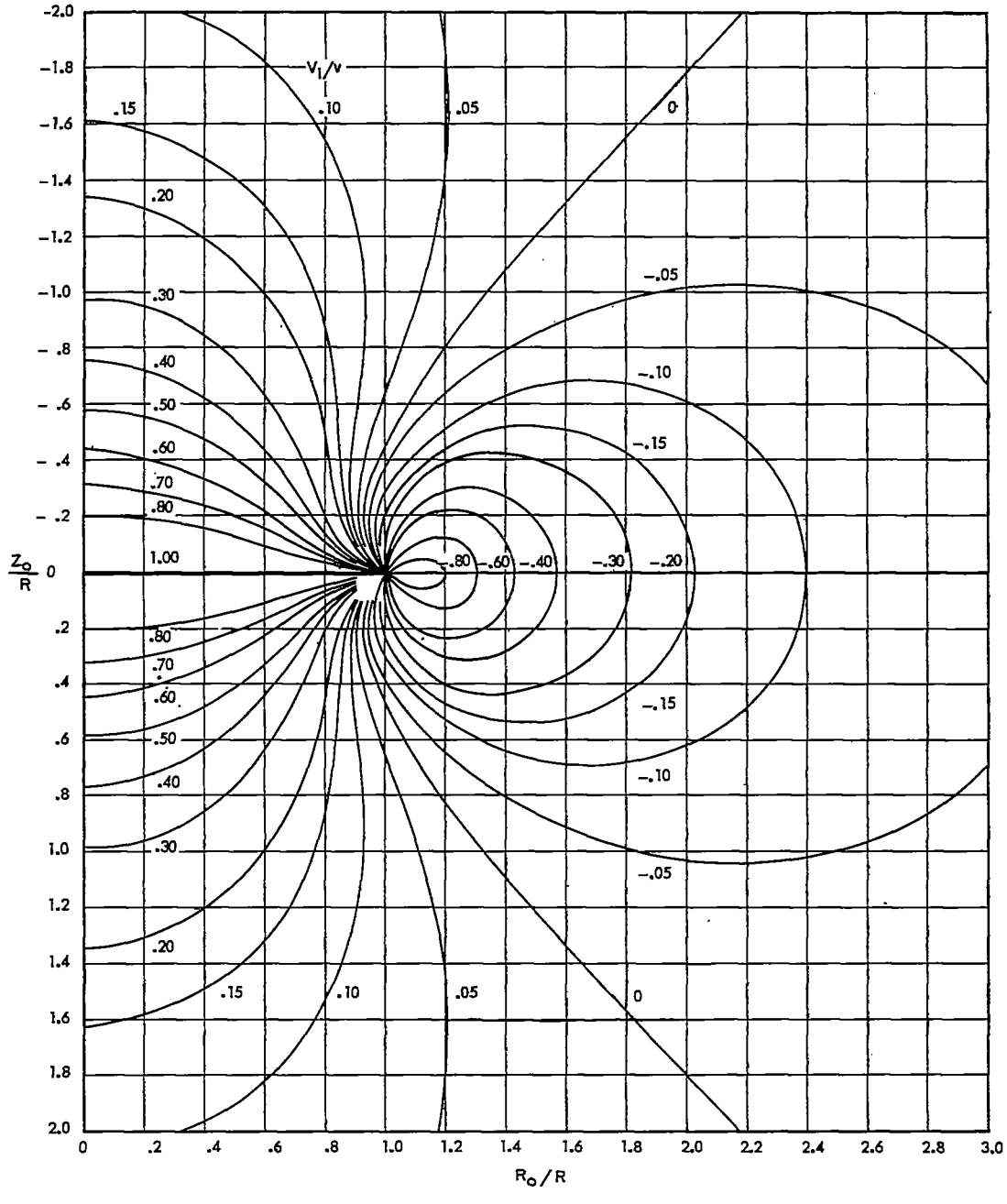
(c) Wake angle = 75.97° or $\chi = \tan^{-1}4$.

Figure 2.- Continued.



(d) Wake angle = 84.29° or $\chi = \tan^{-1}10$.

Figure 2.- Continued.



(e) Wake angle = 90° or $\chi = \tan^{-1}\infty$.

Figure 2.- Concluded.

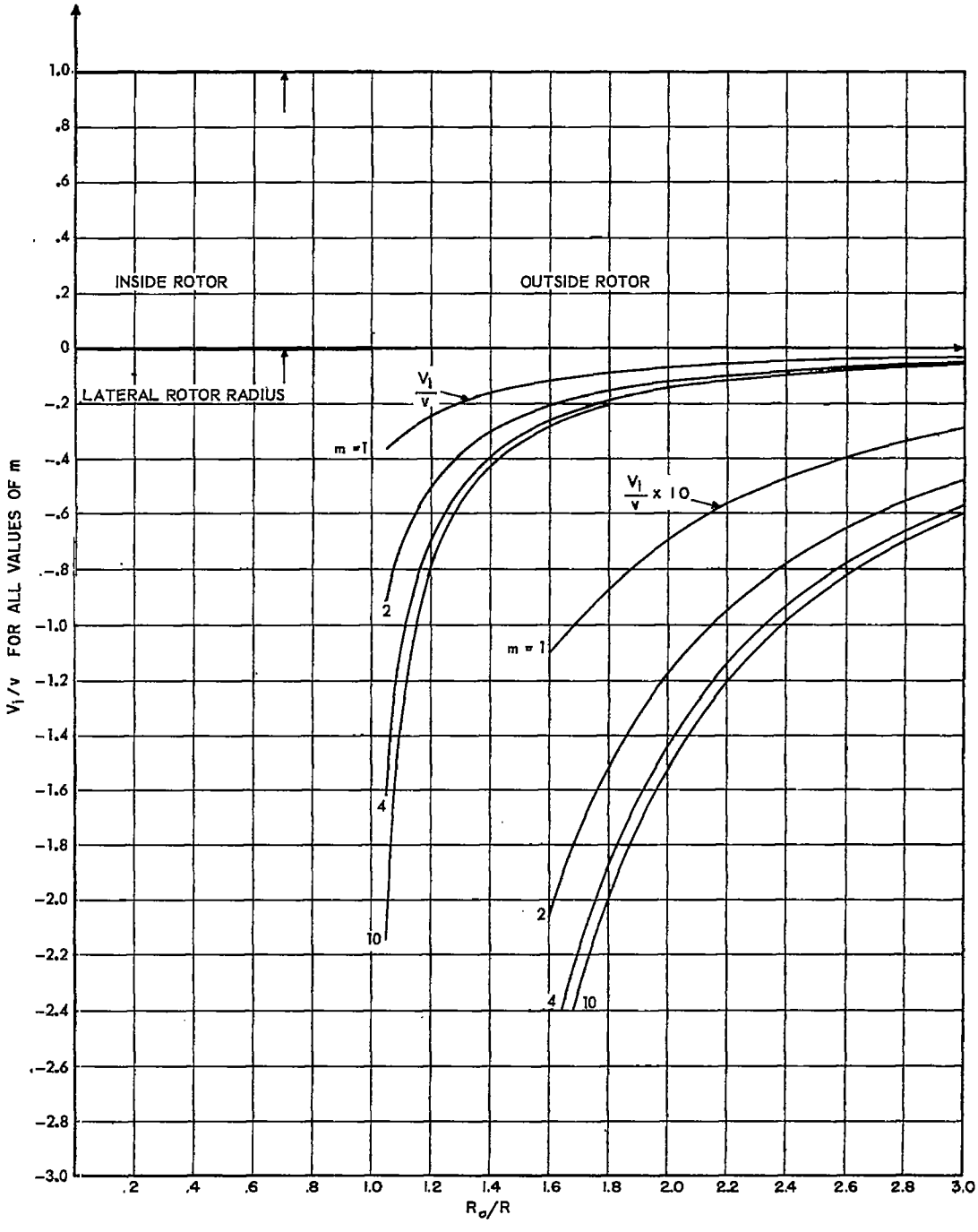


Figure 3.- Values of V_i/v on lateral axis of rotor.

NACA - Langley Field, Va.